Quantum Optimization Algorithms through the lens of Non-Stabilizerness

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Outline

1. Introduction to Non-Stabilizerness
2. Quantum Optimization Algorithms
3. Non-Stabilizerness of Algorithms
4. Concluding Remarks

Pauli Group

Definition (Pauli Group $\mathcal{P}$)

A group containing the 4^n, $n-$qubit tensor products of the identity I and the Pauli matrices $\sigma_x := X$, $\sigma_y := Y$, and $\sigma_z := Z$ is the Pauli group.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

See that $X^2 = Y^2 = Z^2 = 1$ and $Y = iXZ$. To complete the group we allow for phases ± 1, $\pm i$. We can take $\{X, Z\}$ to be the generators of the group.

Quantum Gates

Consider Hadamard, phase and entangling gate $CNOT$,

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, S = \begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}, CNOT = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}. \quad (2)$$

$$HXH^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = Z. \quad (3)$$

Likewise,

$$\begin{aligned} H: &\quad Z \mapsto X, \\ S: &\quad X \mapsto Y, \qquad Z \mapsto Z, \\ CNOT: &\quad X \otimes I \mapsto X \otimes X, \qquad I \otimes X \mapsto I \otimes X, \\ &\quad Z \otimes I \mapsto Z \otimes I, \qquad I \otimes Z \mapsto Z \otimes Z. \end{aligned}$$

Clifford Group

Definition (Clifford Group gates $\mathcal{C}$)

Set of gates that maps elements of Pauli group to another element of Pauli group under conjugation are Clifford group gates. This group is infact generated by $\{H, S, \& \ CNOT\}$.

However, there can be significant number of gates outside of this group.

T-Gate

Consider for instance a T-gate that can be written in matrix representation as,

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}. \tag{4}$$

Non-Clifford beahviour is evident from observing:

$$TXT^{\dagger} = \frac{1}{\sqrt{2}}X + \frac{1}{\sqrt{2}}Y \notin \mathcal{P}. \tag{5}$$

A 'universal gate set' that forms the basis for all possible gates requires non-Clifford resource. In particular $\{H, S, CNOT\} + T$ gate is shown to form a universal gate set [Ref. Nielsen and Chuang, 2010].

Foundational Theorem

Why am I talking about this?

Theorem (Gottesman-Knill, 1998)

Any quantum computer containing only:
a) Clifford group gates, b) measurements of Pauli group operators,
c) Clifford group operations conditioned on classical bits, which may be the results of earlier measurements,
can be perfectly simulated in polynomial time on a probabilistic classical computer.

An underlying fact indicated by the theorem here is: for any kind of quantum computational advantage, entanglement is not enough.

Stabilizer Rényi Entropy

Defined in Ref. [Leone, Oliviero, Hamma, 2022]:

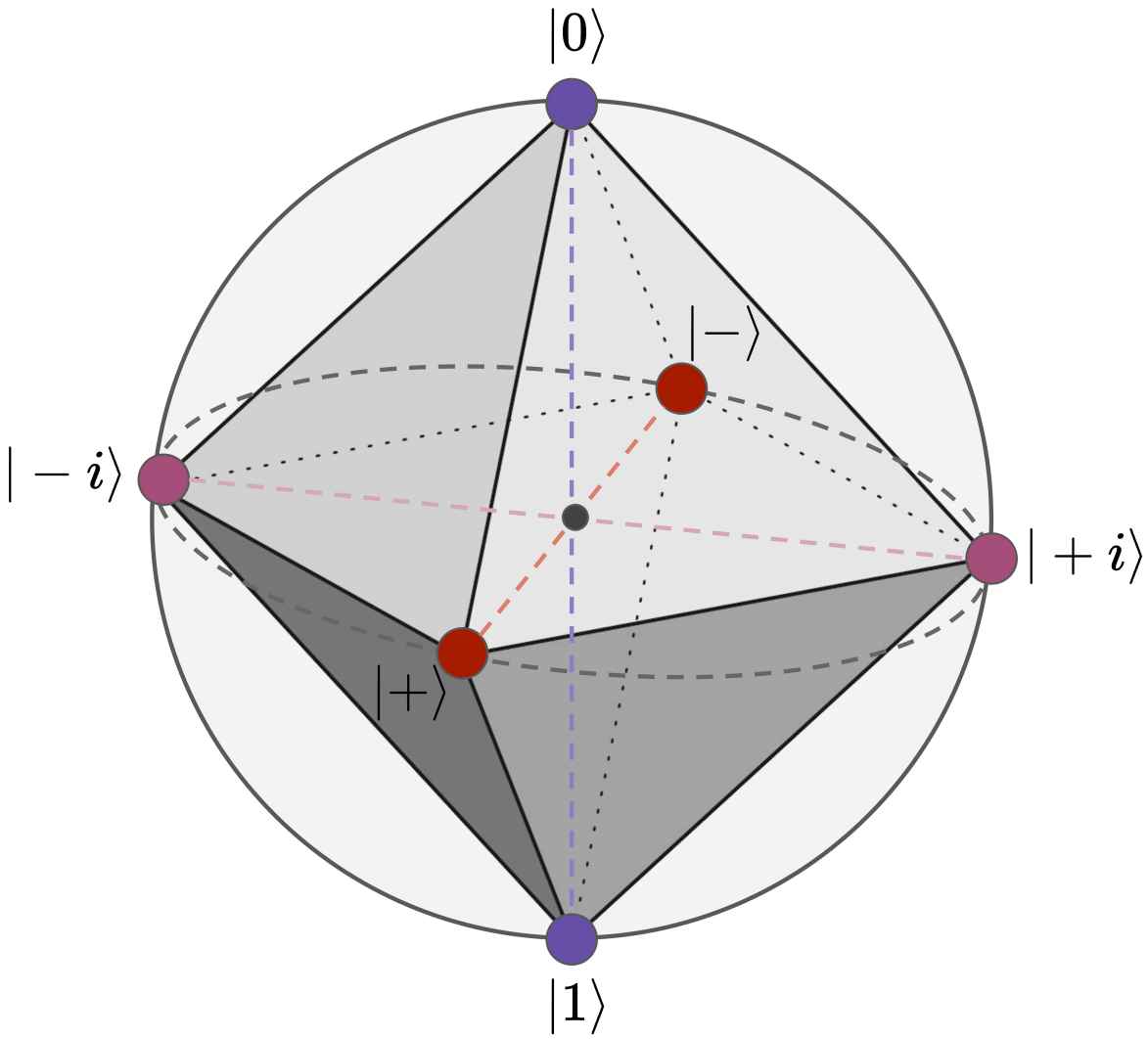
$$M_2(|\psi\rangle) = -\log\left[\frac{\sum_{P\in\mathcal{P}_n} |\langle\psi|P|\psi\rangle|^4}{2^n}\right] \tag{6}$$

quantifies the amount of non-stabilizerness of a quantum state.
We have quantification of quantum complexity for a given state, that is prepared in quantum computer.
E.g.

$$X|0\rangle = |1\rangle, \qquad Y|0\rangle = i|1\rangle, \qquad Z|0\rangle = |0\rangle, \qquad I|0\rangle = |0\rangle$$

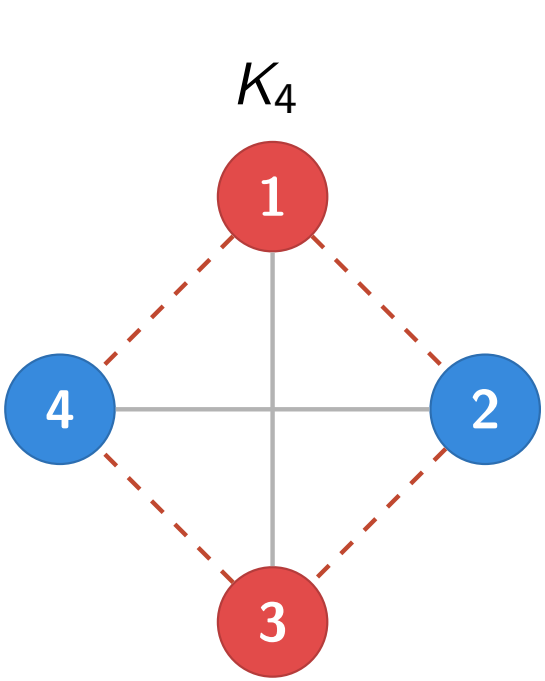
$$M_2(|0\rangle) = -\log_2\left(\frac{|1|^4 + |1|^4}{2}\right) = -\log_2(1) = 0$$

Stabilizer States



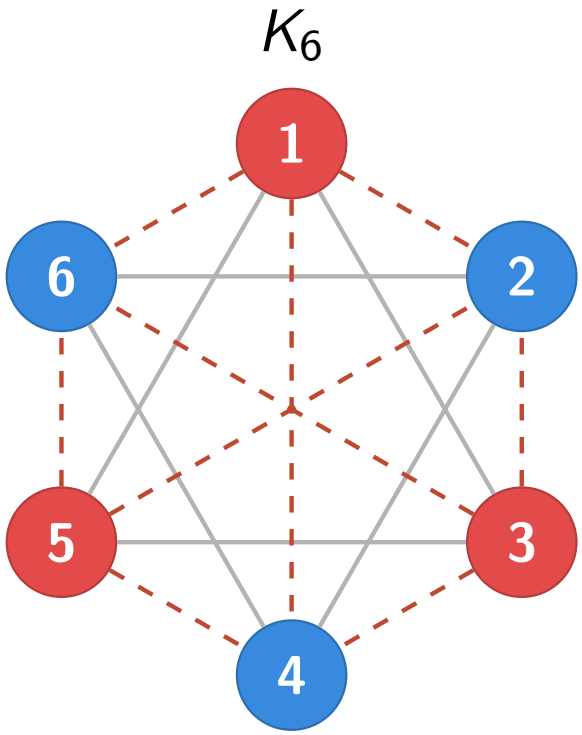
Ref. Pennylane Tutorial

MaxCut on K_4, K_6, K_8



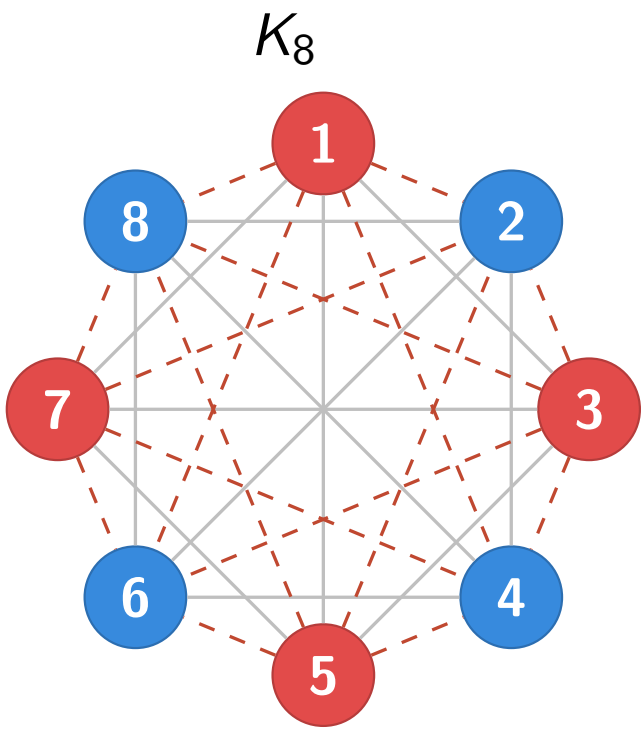
6 edges **4 cut**

$\{1, 3\}$ vs $\{2, 4\}$



15 edges **9 cut**

$\{1, 3, 5\}$ vs $\{2, 4, 6\}$



28 edges **16 cut**

$\{1, 3, 5, 7\}$ vs $\{2, 4, 6, 8\}$

Set A Set B - - - Cut edge ($A \leftrightarrow B$) — Same-set edge

$H = -\sum_{i<j} J_{ij}\, Z_i Z_j,\ J_{ij} \sim \mathcal{N}(0, 1)$

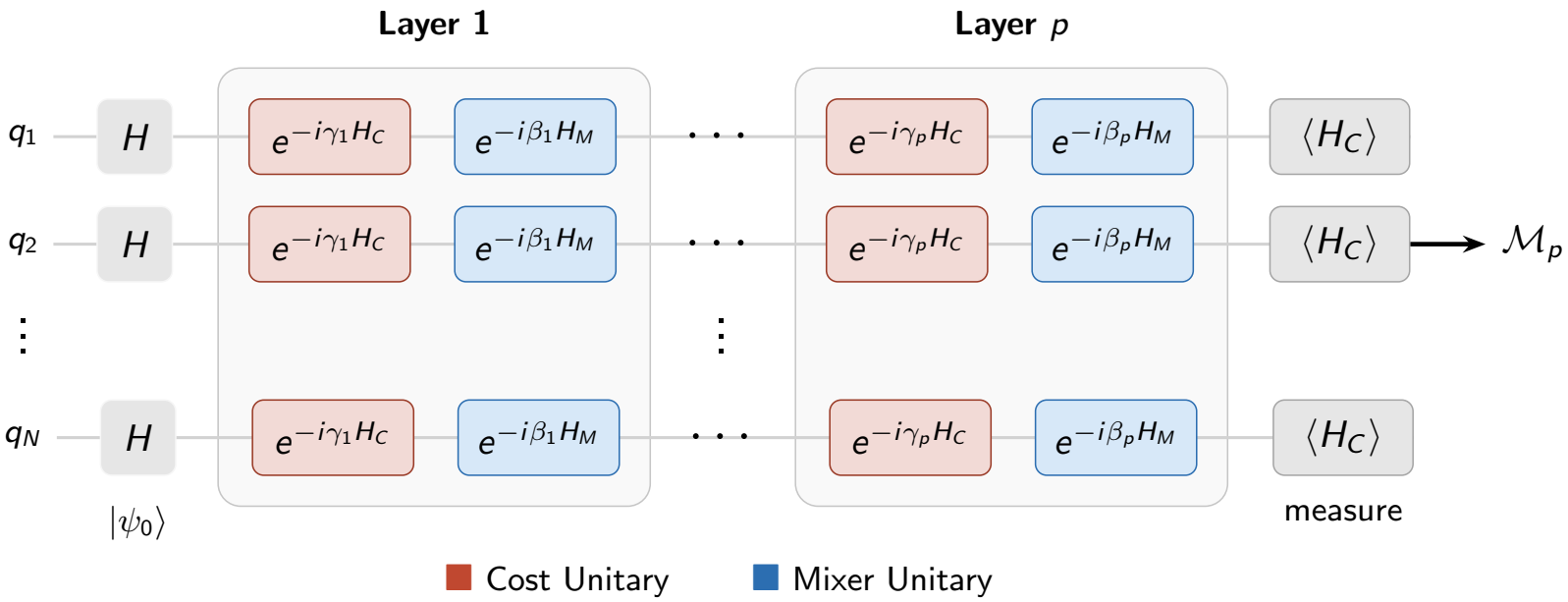
Sherrington Kirkpatrick (SK)-Hamiltonian

We consider an SK-Hamiltonian, with all-to-all random interactions that are randomly sampled from independent Gaussian distributions, in agreement with the setup in Ref.[Capecci et. al. 2025]. We consider

$$\hat{H}_{\mathrm{SK}} = \sum_{i} \sum_{i<j} J_{ij} Z_i Z_j + \sum_{i} h_i Z_i, \tag{7}$$

where the coefficients J_{ij} are randomly chosen from a normal distribution with unit variance, Z_i is the Pauli-Z operator acting on the $i-$th qubit. To remove degeneracies in the ground state, we add bias magnetic field terms $\hat{H}_{\mathrm{bias}} = h_i Z_i$, where coefficients h_i are chosen randomly from a normal distribution with a variance of 0.3.

QAOA Circuit Schematic



QAOA

1. define cost Hamiltonian H_C to be the SK-Hamiltonian with bias term, whose ground state encodes the solution to the optimization algorithm.
2. define the mixer Hamiltonian H_M, whose ground state is easy to construct.
3. prepare the quantum circuits: cost layers given by $e^{-iH_C\gamma}$, and mixer layers given by $e^{-iH_M\beta}$.
4. choose parameter p to build a layered circuit

$$U(\vec{\gamma},\vec{\beta}) = \prod_{k=1}^{p} e^{-iH_M\beta_k} e^{-iH_C\gamma_k}. \tag{8}$$

5. prepare initial state, apply $U(\vec{\gamma},\vec{\beta})$ and apply suitable classical optimization protocol.
6. following the completion of the optimization process, perform measurements and obtain the approximate solution to the optimization problem.

ADAPT-QAOA

1. define the mixer pool $\{A_k\}$, where $A_k = A_k^\dagger$, and prepare initial state as in standard QAOA.
2. for the current layerwise ansatz, measure the energy gradient

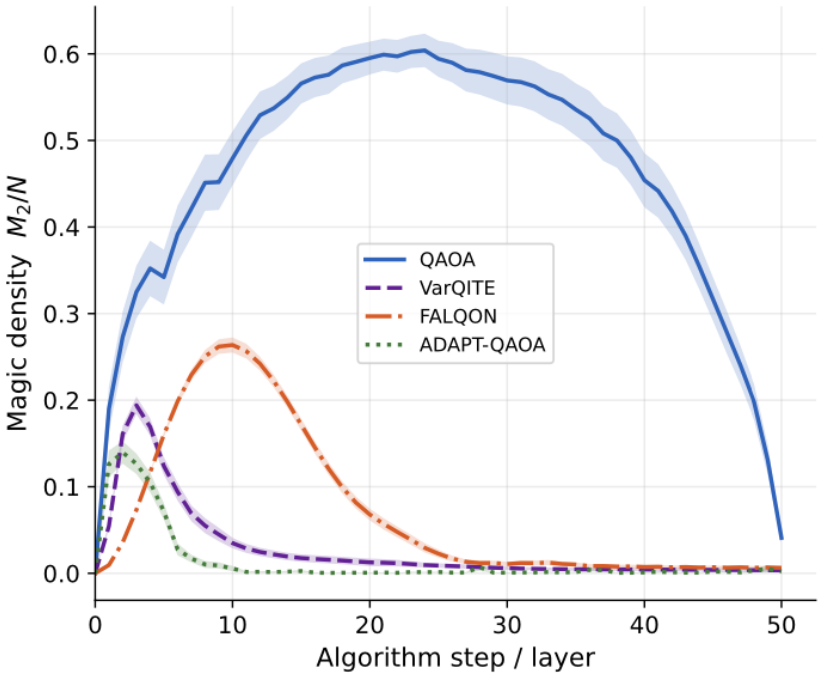
$$\langle\psi^{(k-1)}|e^{iH_C\gamma_k}[H_C, A_j]e^{-iH_C\gamma_k}|\psi^{(k-1)}\rangle, \tag{9}$$

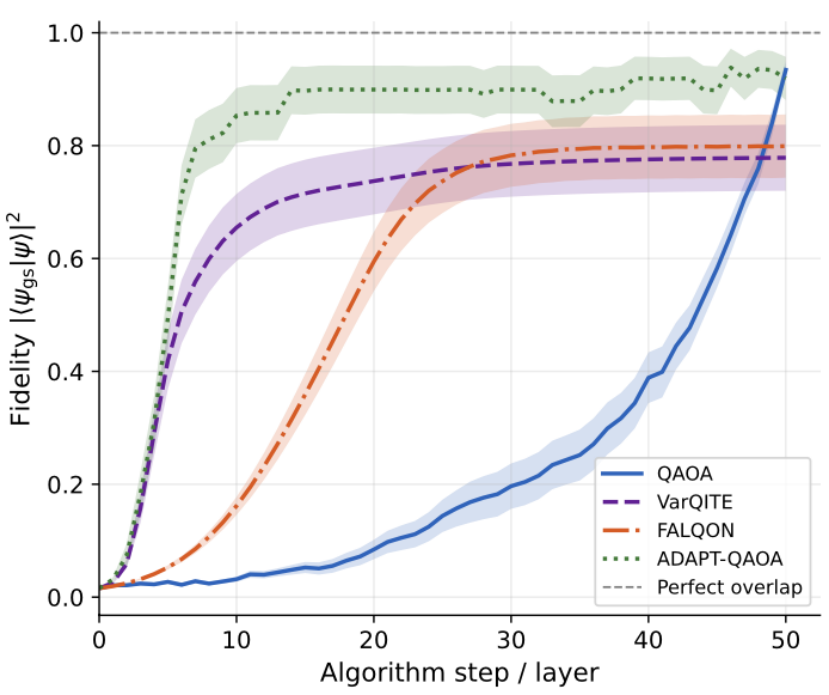
   where the new variational parameter γ_k is set to a fixed value.
3. adjust the ansatz by adding the operator $\{A_k\}_{\max}$, which gives the largest magnitude of the energy gradient to give:

$$|\psi^{(k)}\rangle = e^{-iA_k^{\max}\beta_k}\ e^{-iH_C\gamma_k}|\psi^{(k-1)}\rangle, \tag{10}$$

   where β_k's are the variational parameters.
4. optimize all the parameters in the current ansatz, i.e., β_m, γ_m for $m = 1, \ldots k$ to minimize the expectation value of the cost Hamiltonian and return to step 2.

Non-Stabilizerness of Algorithms (n=6)



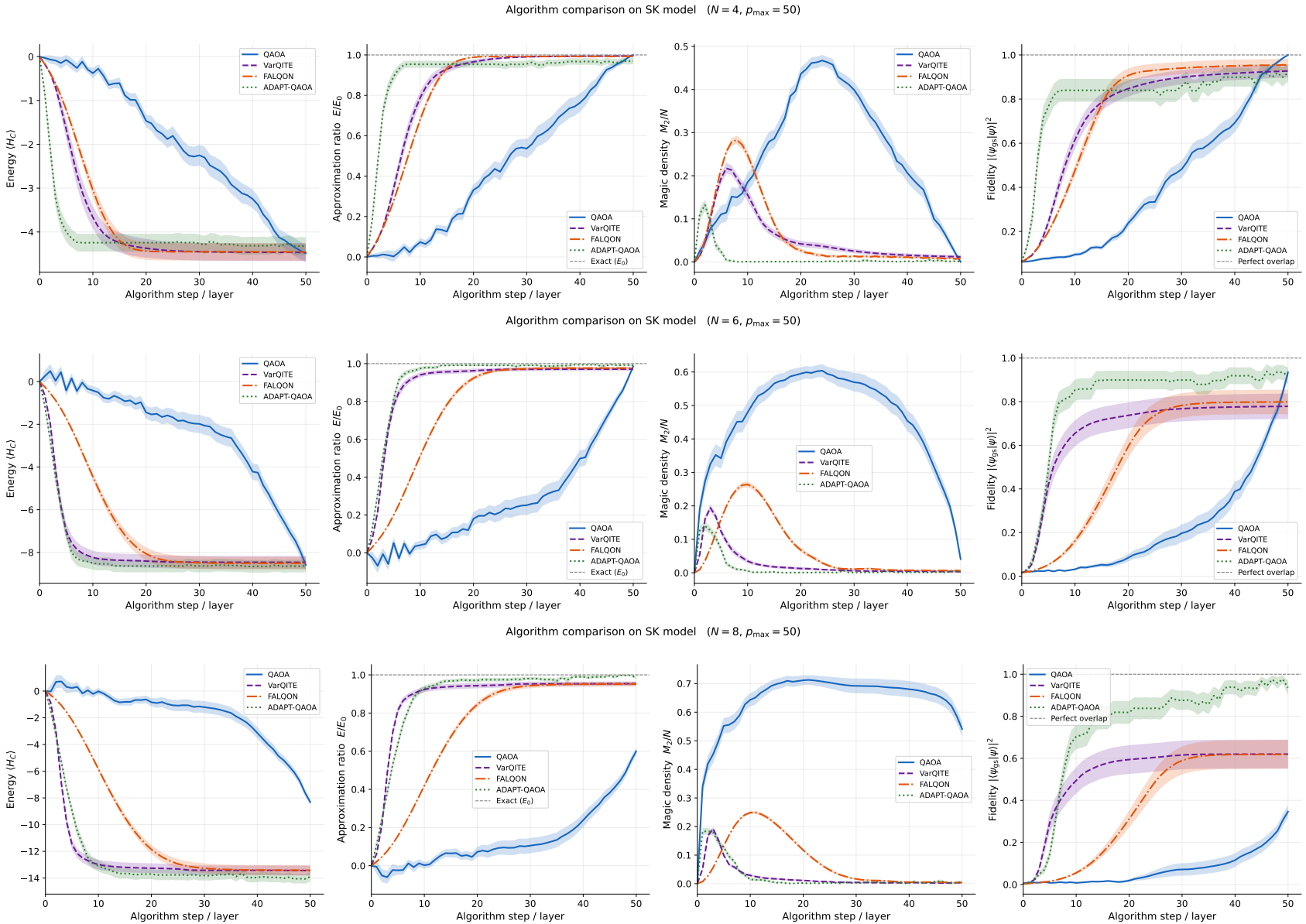


Concluding Remarks

1. Quantum advantage requires entanglement as well as non-stabilizerness.
2. Non-Stabilizerness quantified by 2-Stabilizer Rényi entropy is a good measure to understand complexity of the quantum algorithms.
3. QAOA is much more computationally expensive than other algorithms, and traverses a path away from stabilizer manifold.
4. An interesting tool to study quantum mechanical system.

Thank You!

Backup slides-Non-Stabilizerness of Algorithms



Backup slides-FALQON

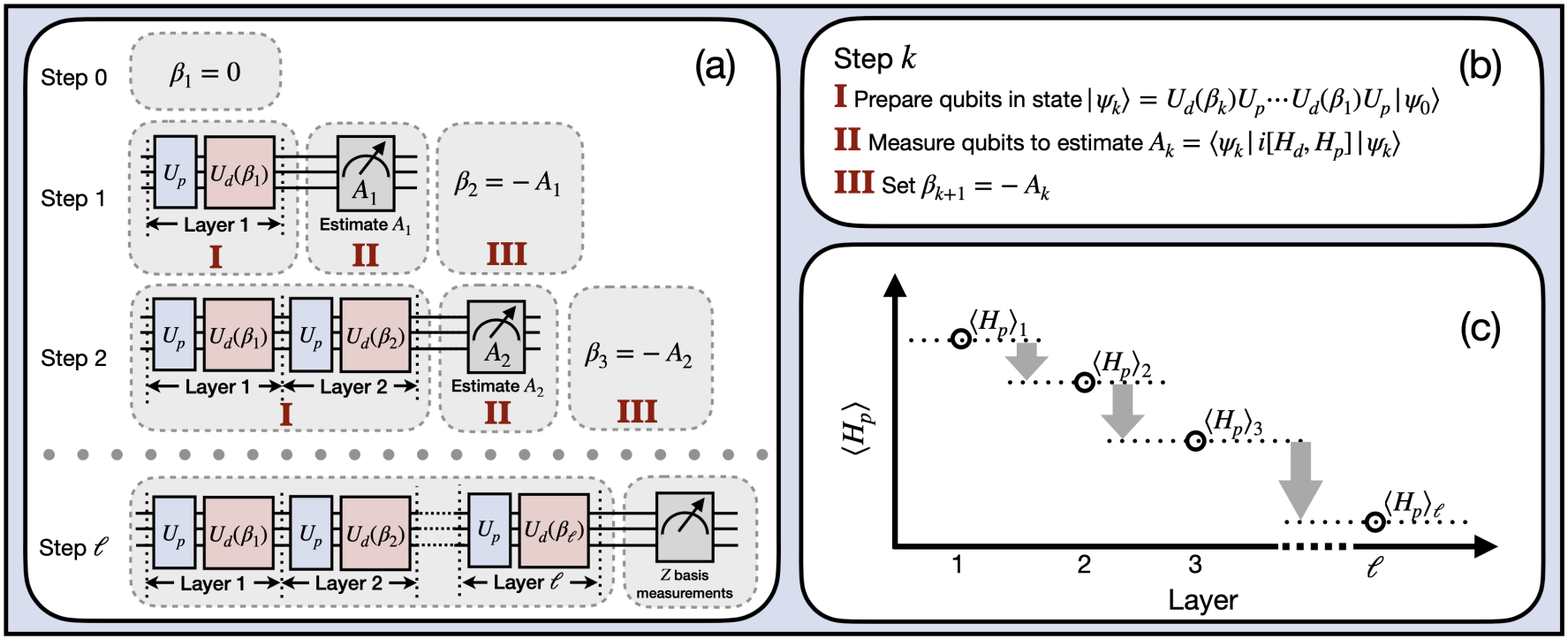


Figure: FALQON schematics (Taken from Magann et. al, 2023)

Backup slides-VarQITE

Build a parametrized quantum circuit which encodes the state $|\psi(\tau)\rangle$, via a set of variational parameters $\Theta(\tau) = (\theta_1(\tau), \theta_2(\tau), \ldots, \theta_N(\tau))$. Using McLachlan's variational principle an optimal approximation for the exact imaginary time evolved state can be found within the ansatz manifold with minimization of the distance:

$$\delta \Big|\Big| \Big(\frac{\partial}{\partial \tau} + \hat{H} - \langle \hat{H} \rangle_{\Theta(\tau)} \Big) |\phi(\Theta(\tau))\rangle \Big|\Big| = 0, \tag{11}$$

where $||\rho|| := tr[\sqrt{\rho \rho^\dagger}]$ is the trace-norm of a state and $\langle \hat{H} \rangle_{\Theta(\tau)} := \langle \phi(\Theta(\tau)) | \hat{H} | \phi(\Theta(\tau)) \rangle$.

Backup slides - VarQITE

The dynamics of the parameter space is governed by the differential equation,

$$\sum_j A_{ij}\dot{\theta}_j = C_i \tag{12}$$

where,

$$A_{ij} = \Re\left(\left\langle \frac{\partial\phi(\tau)}{\partial\theta_i}\middle|\frac{\partial\phi(\tau)}{\partial\theta_j}\right\rangle\right) := \Re\left(\langle\partial_i\phi(\tau)|\partial_j\phi(\tau)\rangle\right),$$

$$C_i = \Re\left(\left\langle \frac{\partial\phi(\tau)}{\partial\theta_i}\middle|\hat{H}\middle|\phi(\tau)\right\rangle\right) := \Re\left(\langle\partial_i\phi(\tau)|\hat{H}|\phi(\tau)\rangle\right).$$