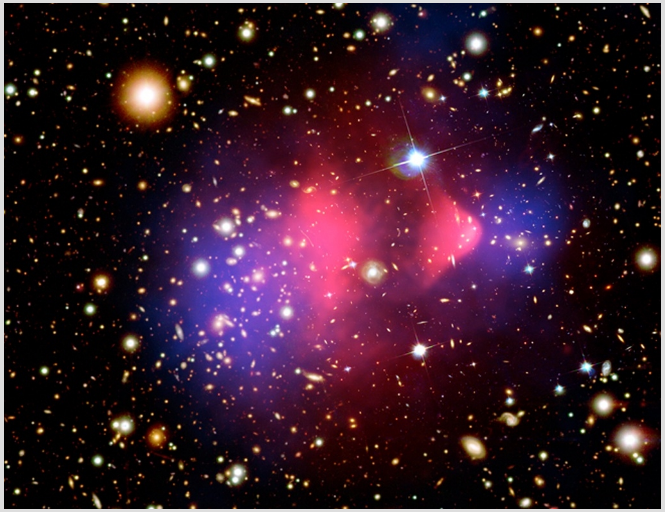
Review of Mathematics, Numerical Factors, and Corrections for Dark Matter Experiments Based on Elastic Nuclear Recoil

Miguel Angel Soto March 25th

Introduction

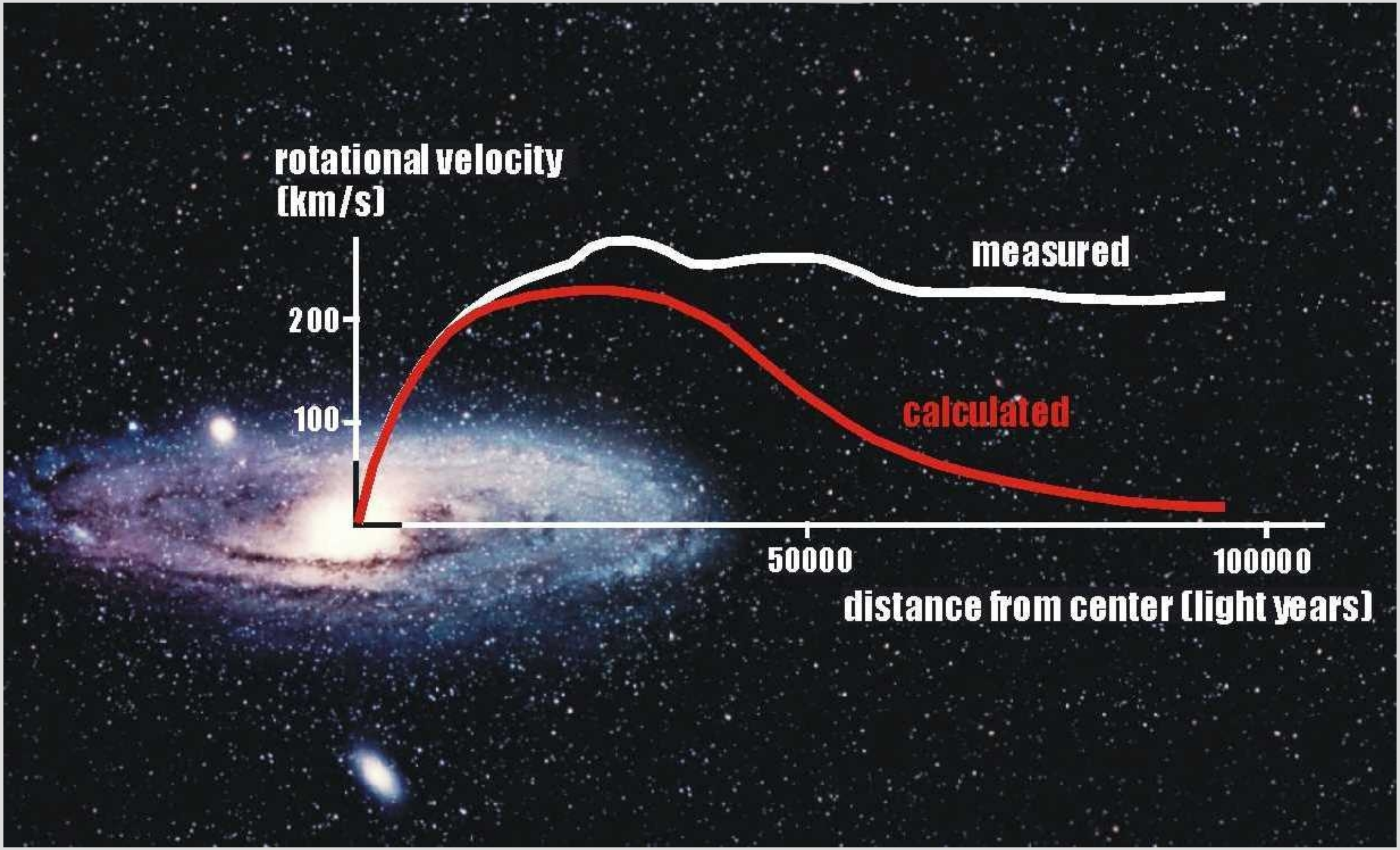
First observations of an invisible matter made by Zwicky in the 1930’s and then ignored until observations by Rubin in 1970’s

What it is? May take a while



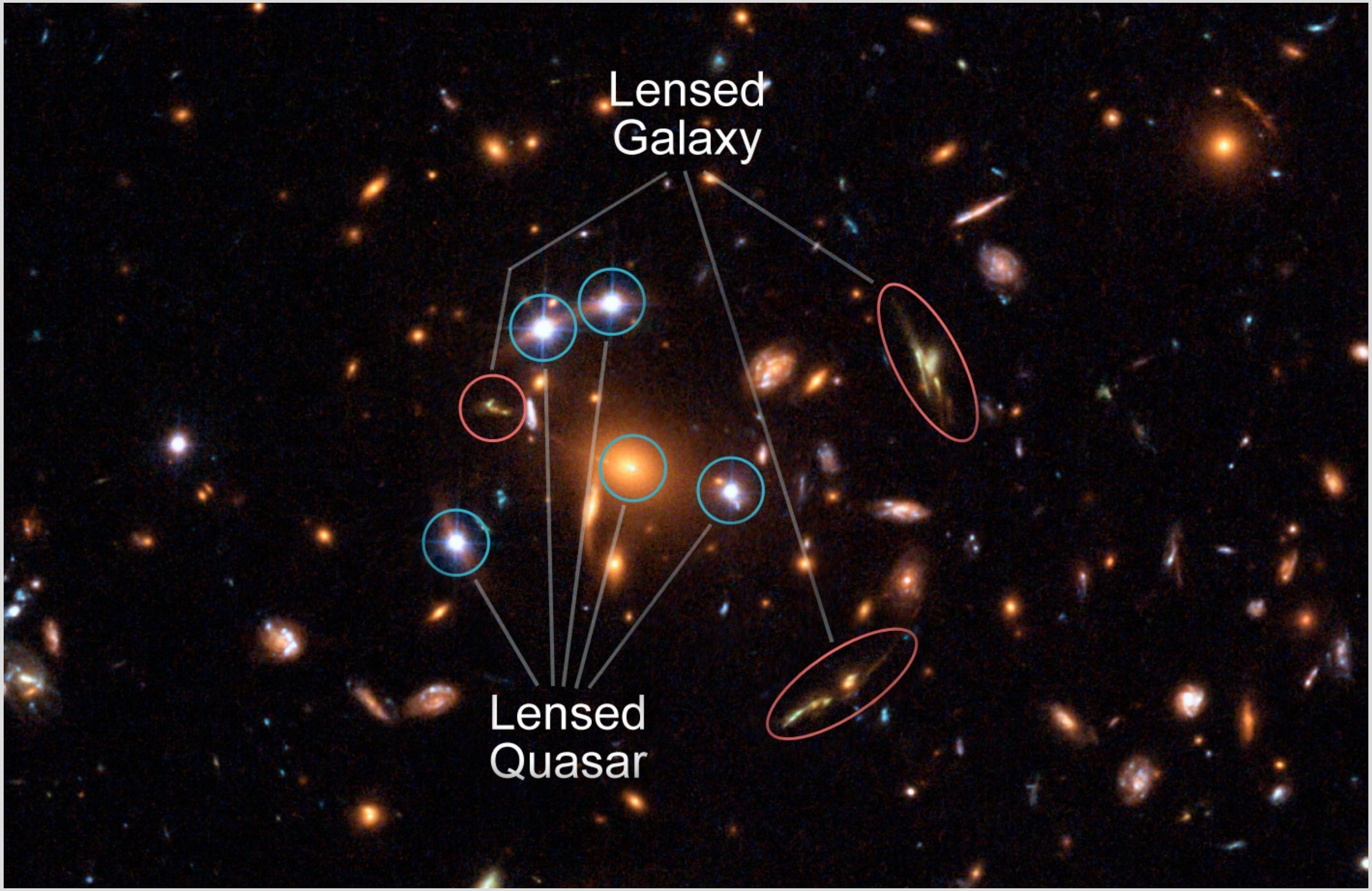
Evidences

- **Rotational curves**



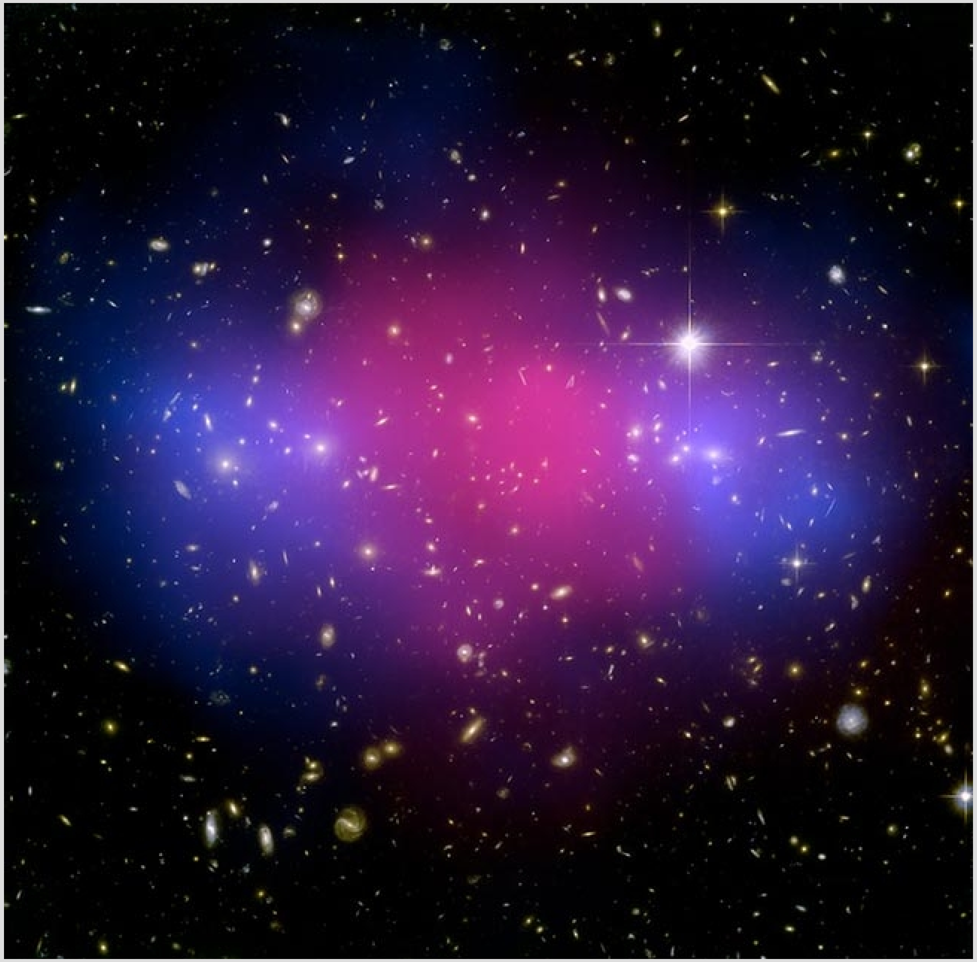
Evidences

- **Gravitational Lensing**

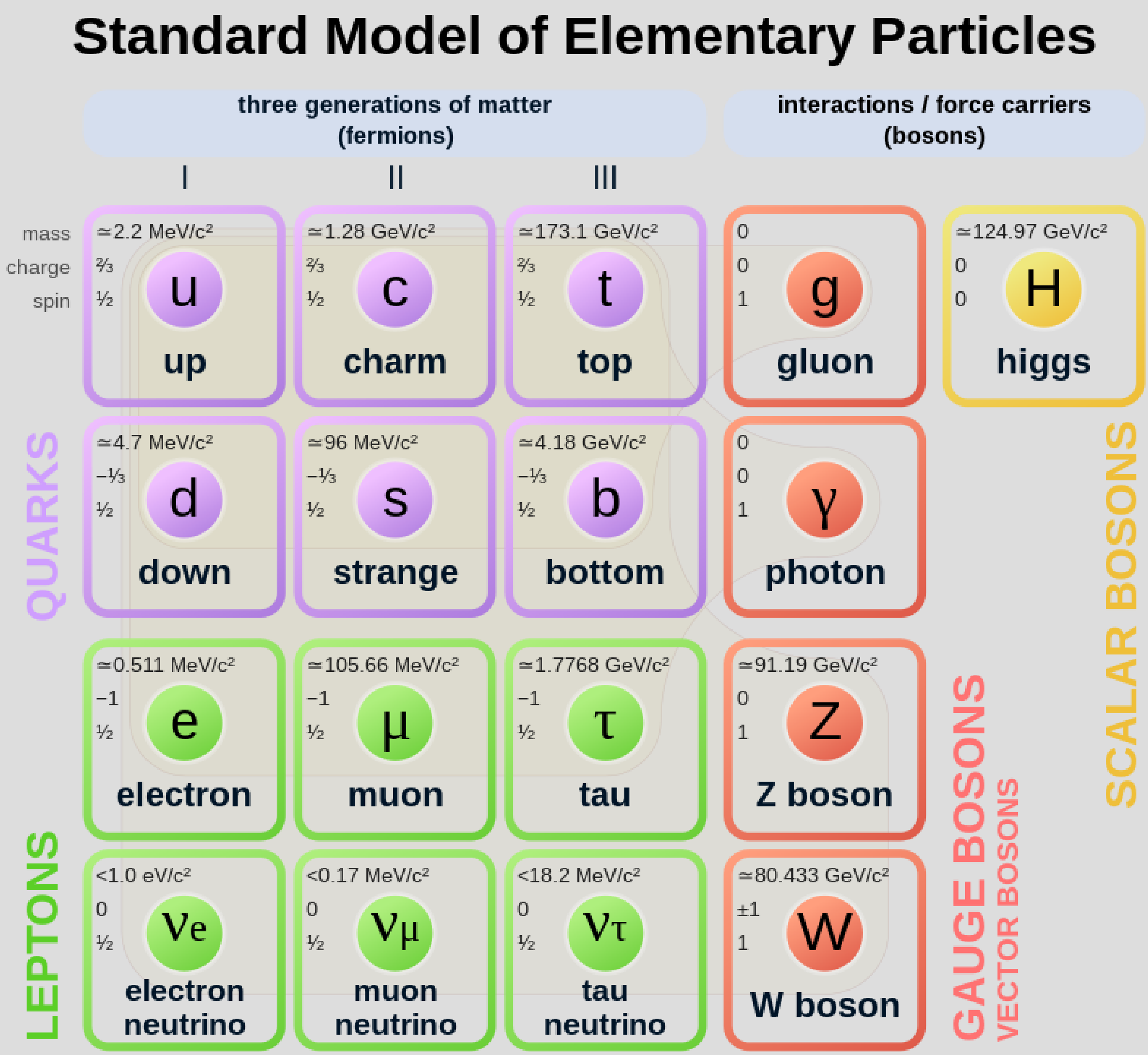


Evidences

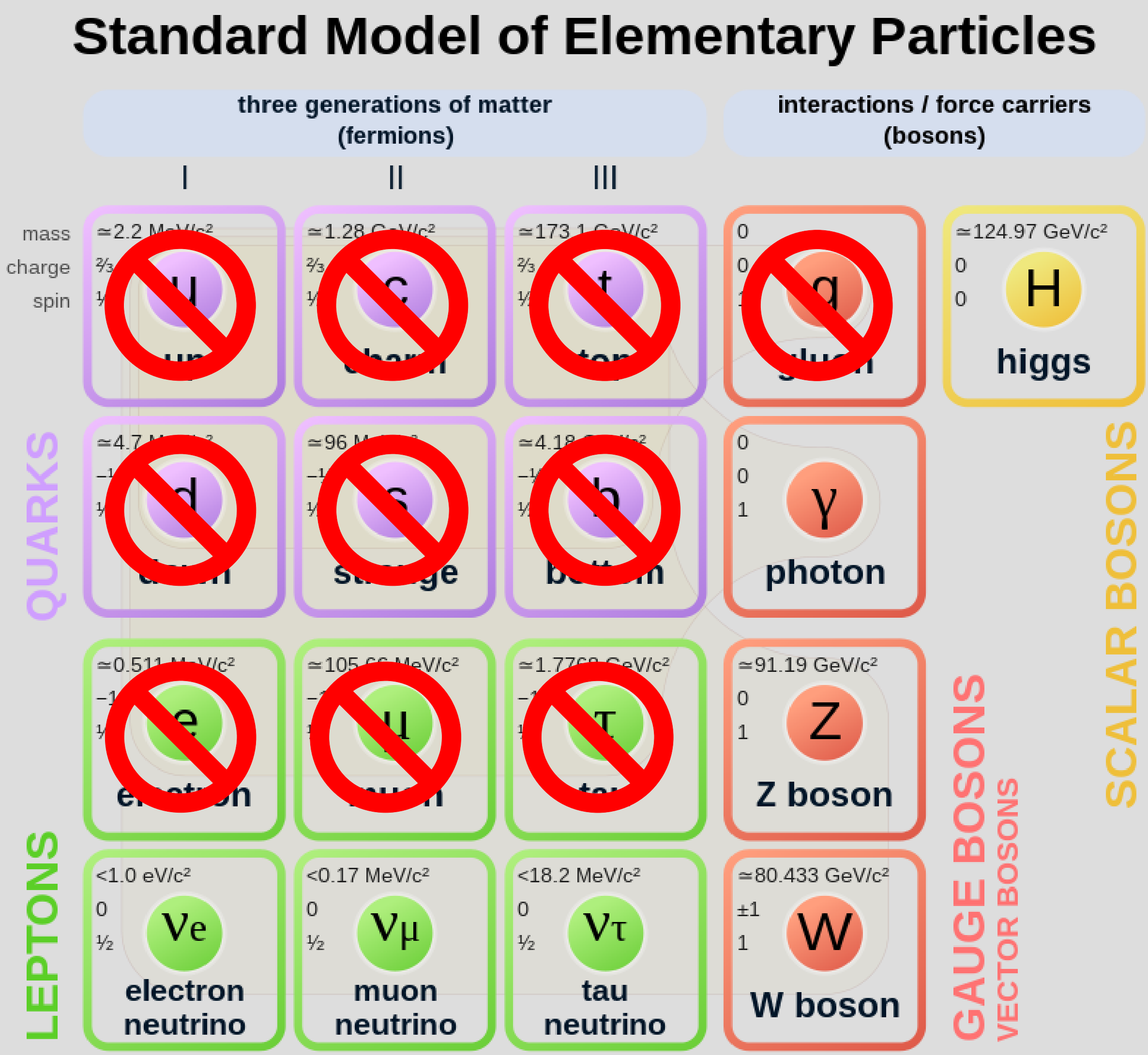
- **Bullet Cluster**



Standard Model Possibility

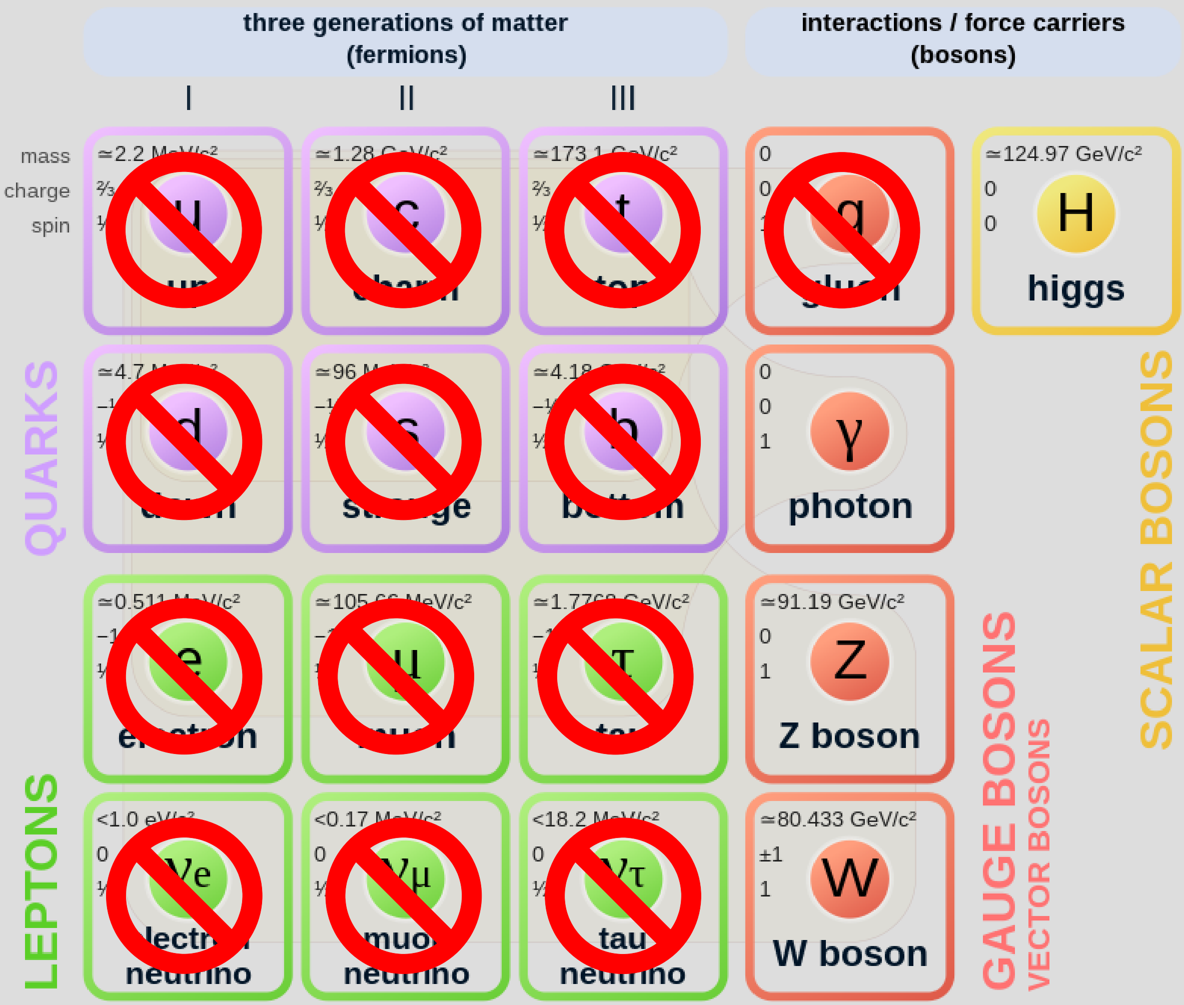


Standard Model Possibility

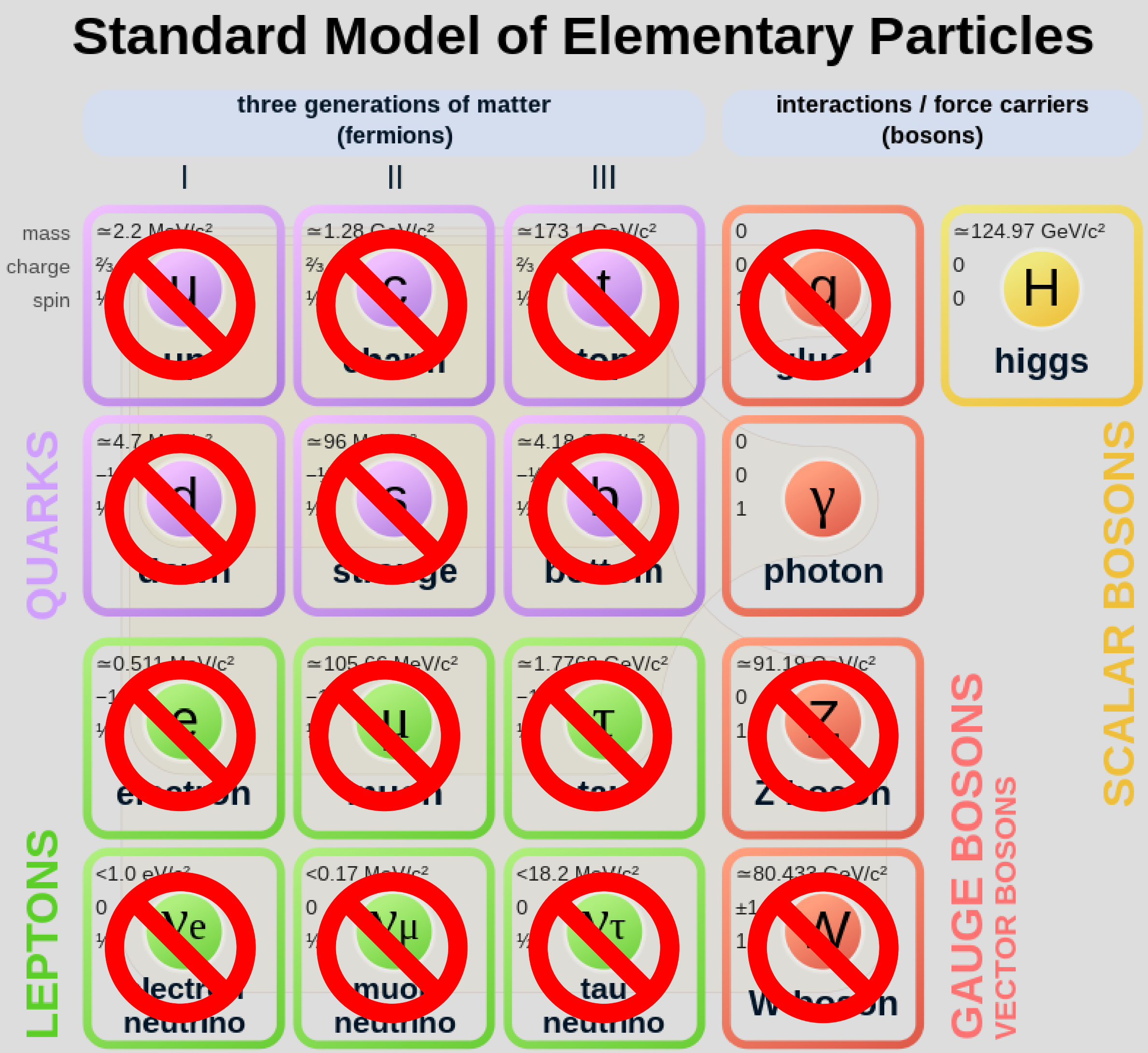


Standard Model Possibility

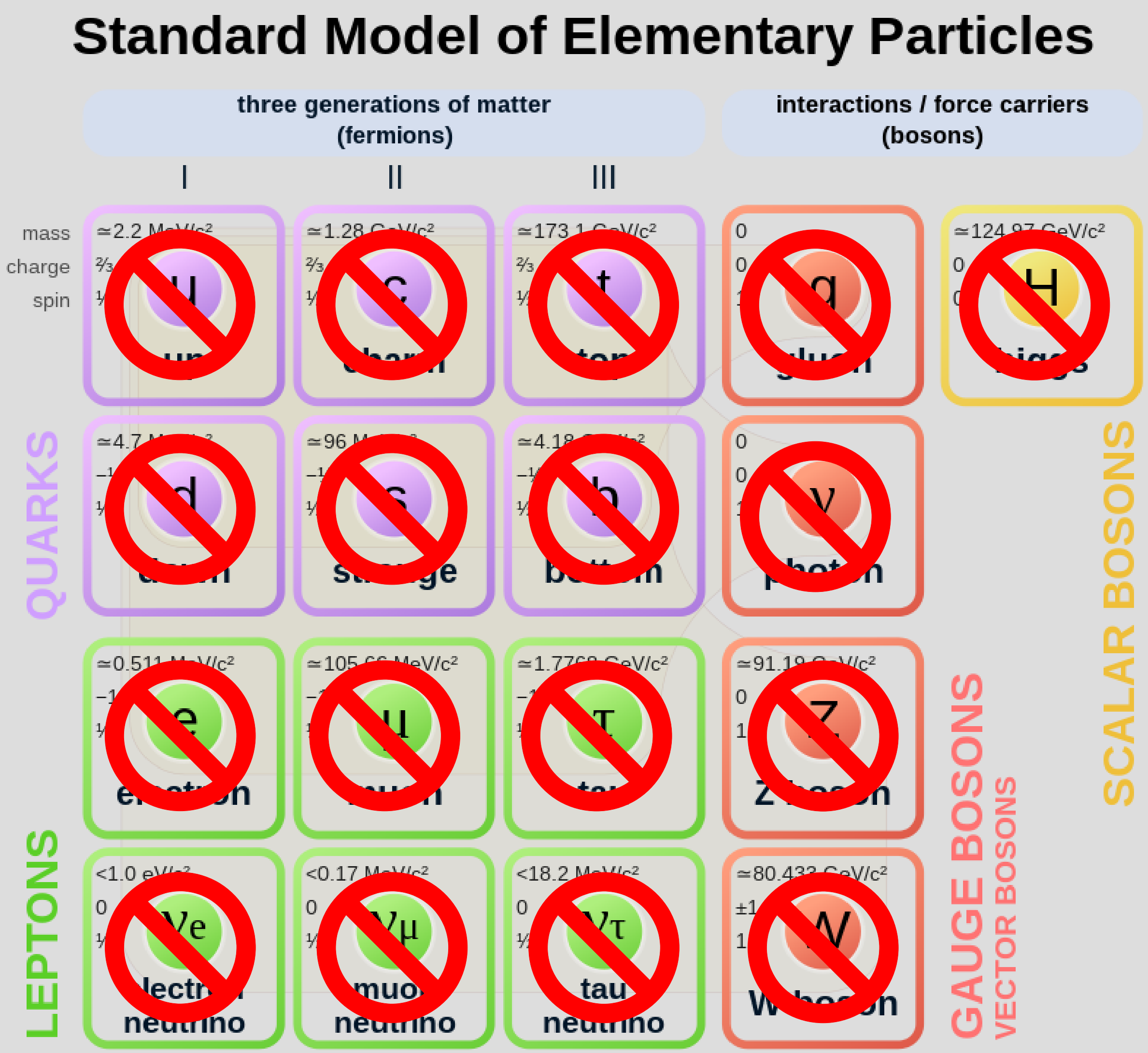
Standard Model of Elementary Particles



Standard Model Possibility



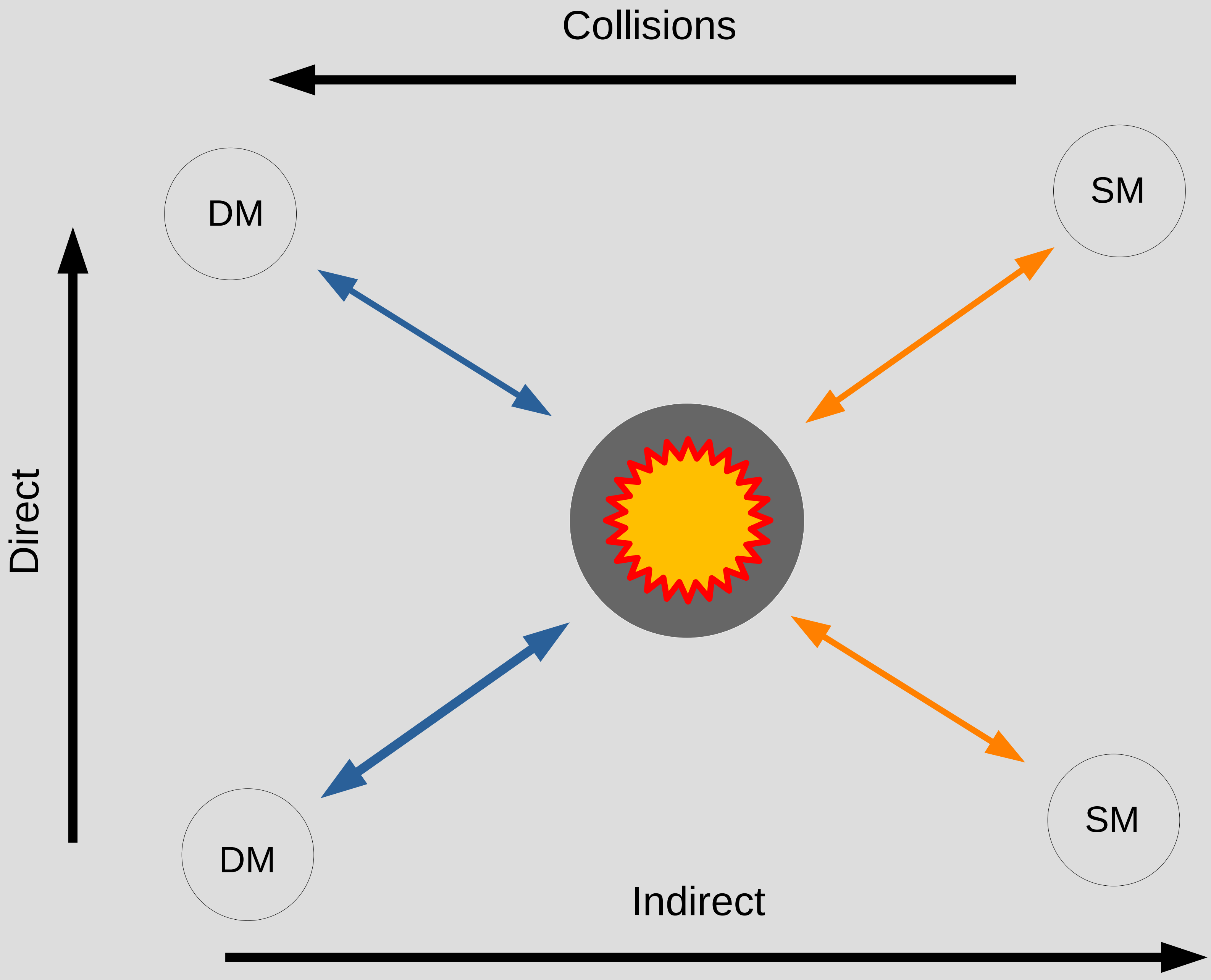
Standard Model Possibility



Candidates

- Axions
- Sterile Neutrinos
- Weakly Interactive Massive Particles (WIMPS)
- Gravyton
- Not particles at all?

Detection



Experimental Efforts Direct Detection

Nuclear collisions

- Differential of event rate with respect to recoil energy:

$$\frac{dR}{dE}\Big|_{\text{observed}} = R_0 f_A S(E) F^2(E) I_c$$

- $S(E)$: Modified spectral function

$F(E)$: Form factor correction (Due to finite size of nucleus)

I : Interaction function (Different for spin-dependent and spin-independent factor)

f_A : Relative efficiency

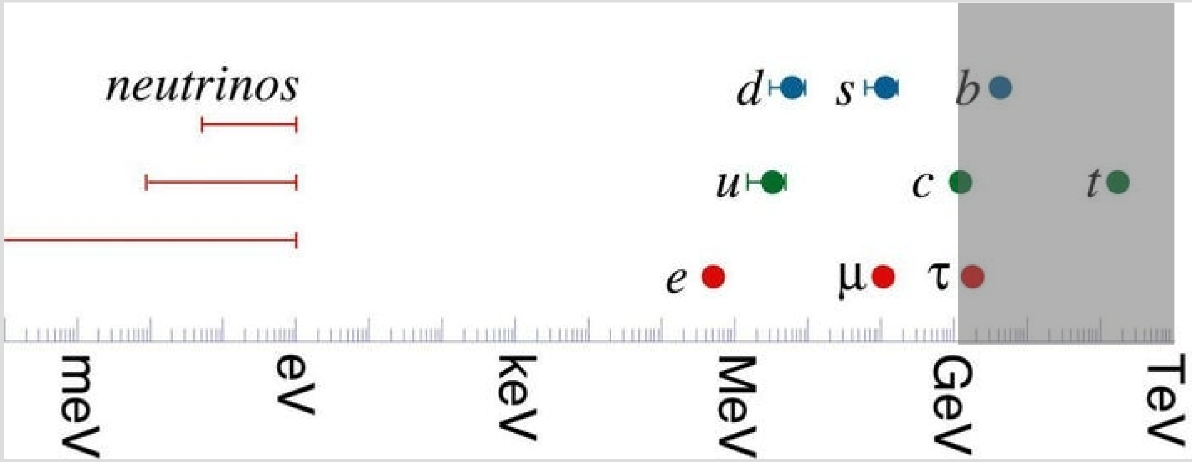
- For DM particles coming from the center of our galaxy

If detector is stationary in galaxy, simple model

$$\frac{dR}{dE_R} = \frac{R_0}{E_o r} e^{-E_R/E_0 r}$$

Galactic velocities of $10^{-3}c$, masses of $M_D = 10 - 1000 GeV c^{-2}$

result $E_R = 1 - 100 keV$



Modified Spectral Function

- Particle Density and velocity distribution

$$dn = \frac{n_0}{k} f(v, v_E) d^3 v$$

Event Rate per unit mass

$$dR = \frac{N_0}{A} \sigma v dn \;\; \Rightarrow \;\; R = \frac{N_0}{A} \sigma_0 n_0 < v >$$

$$= R_0 \frac{k_0}{k} \frac{1}{2\pi 4 v_0^4} \int v f(v, v_E) d^3 v$$

Differential form mostly used

Form Factor Correction

- $F(qr_n)$ represents the falling of cross section with increasing momentum transfer

$$\sigma(qr_n) = \sigma_0 F^2(qr_n)$$

Some easy models to consider

$$F^2(qr_n) = [\sin(qr_n)/qr_n]^2\,, \quad F^2(qr_n) = \left[\frac{3(\sin(qr_n) - qr_n\cos(qr_n))}{(qr_n)^3}\right]^2$$

Spherical shell and solid sphere

Detector Response Corrections

In detectors, the effective nuclear recoil energy is less than the true value. This is called "relative efficiency" f_n

The rates in the "visible energy" $E_R = E_v / f_n$

$$\frac{dR}{dE_R} = f_n \left(1 + \frac{E_R}{f_n}\frac{df_n}{dE_R}\right)\frac{dR}{dE_v}$$

For ionization detectors

$$f_n = \frac{kg(\epsilon)}{1 + kg(\epsilon)}$$

$$\epsilon = 11.5 E_R Z^{-7/3}$$

$$k = 0.133 Z^{2/3} A^{1/2}$$

Interaction Factor

- For low momentum transfer $qr_n << 1$, would add scattering amplitude A in phase to give $\sigma \propto A^2$.

  For single nucleon, then $I_c \equiv A^2$

  This in practice becomes more complicated, i.e for neutrinos, it becomes $I_c \sim (A - Z)^2$

  When spin dependency shows up, then scattering amplitude changes sign with direction.

  Nuclei with odd number of protons or neutrons are allowed

Summary

- The differential energy spectrum for nuclear recoils reduces to

$$\frac{dR}{dE}\Big|_{\mathrm{observed}} = R_0 f_A S(E) F^2(E) I_c$$

  Each term making a contribution

  Mostly experimental and approximations from observations

- Running Experiments:
  - ATLAS (Large Hadron Collider)
  - Axion Dark Matter Experiment