# Magnetohydrodynamic Waves in the Presence of Magnetic Monopoles

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# Overview

- Not my field of expertise!
- Theory-heavy!
- Speculative!

# Motivation - Duality

Vacuum

$$\begin{cases} \nabla \cdot \boldsymbol{E} = 0 \\ \nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \end{cases}$$

# Motivation - Duality

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With Sources

$$\begin{cases} \nabla \cdot \boldsymbol{E} = \rho_e \\ \nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{B} = 0 \\ \nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\boldsymbol{J}_e \end{cases}$$

# Modified EM Equations

$$\begin{cases} \nabla \cdot \boldsymbol{E} = 4\pi\rho_e \\ \nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} - \frac{4\pi}{c}\boldsymbol{J}_m \\ \nabla \cdot \boldsymbol{B} = 4\pi\rho_m \\ \nabla \times \boldsymbol{B} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\boldsymbol{J}_e \end{cases}$$

$$\boldsymbol{F} = q_e\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) + q_m\left(\boldsymbol{B} - \frac{\boldsymbol{v}}{c} \times \boldsymbol{E}\right)$$

# What is a Plasma, Really?

- A macroscopic system of charge carriers that is dominated by *collective*, *electromagnetic* interactions
  - A plasma should be composed of multiple kinds of charge carriers (species), such as electrons and ions
  - Collective interactions: meaning waves and wave-like interactions are much more prominent than collisional interactions
  - Electromagnetic interactions: Charges are strongly coupled to the electromagnetic fields
- Ideal plasmas are quasi-neutral; negligible net charge.

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_s}{e^2 m_s}} \ll \ell \quad N_D = \frac{4}{3}\pi\lambda_D^3 n_s \gg 1 \quad \omega_{pl}\tau_{coll} \gg 1$$

# What is Magnetohydrodynamics (MHD)?

- In statistical physics, we study the evolution of the particle distribution function (PDF) and collisional effects.
- In the hydrodynamical framework, we replace the particle description with a fluid description.
  - I.e., rather than talking about electrons and an electron distribution, we have an electron fluid.

# How to derive the MHD Equations

PDF ↓ Force ↓

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\boldsymbol{F}}{m} \cdot \frac{\partial f}{\partial \boldsymbol{v}} = \left(\frac{\delta f}{\delta t}\right)_{coll}$$

$$\boldsymbol{F} = q_e \left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right) + q_m \left(\boldsymbol{B} - \frac{\boldsymbol{v}}{c} \times \boldsymbol{E}\right)$$

# How to derive the MHD Equations

$$\int v^m \left( \frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\boldsymbol{F}}{m} \cdot \frac{\partial f}{\partial \boldsymbol{v}} \right) d^3 v = \int v^m \left( \frac{\delta f}{\delta t} \right)_{coll}$$

$$\boldsymbol{F} = q_e \left( \boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B} \right) + q_m \left( \boldsymbol{B} - \frac{\boldsymbol{v}}{c} \times \boldsymbol{E} \right)$$

# How to derive the MHD Equations

$$m = 0$$

Continuity Equation

Number density

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{u}) = S$$

$$m = 1$$

Momentum Equation

$$\frac{\partial}{\partial t}(\rho\boldsymbol{u}) + \nabla \cdot (\rho\boldsymbol{u}\boldsymbol{u}) = -\nabla \cdot \boldsymbol{P} + q_e n\left(\boldsymbol{E} + \frac{\boldsymbol{u}}{c} \times \boldsymbol{B}\right) + q_m n\left(\boldsymbol{B} - \frac{\boldsymbol{u}}{c} \times \boldsymbol{E}\right) + \left(\frac{\delta \boldsymbol{M}}{\delta t}\right)_{coll}$$

Mass density

There exists an energy equation (m = 2), but I'll spare you the mess.

# Possible Fluids

Now that we have this model, what kinds of fluids can we make?

- Only magnetic charges?
  - Trivial. A duality transformation can transform the fields and sources so that we recover the purely electric case, which recovers standard MHD.
- Particles with both electric and magnetic charge?
  - Possibly interesting, but physically unmotivated since we haven't observed a particle with both kinds of charge, let alone any magnetic charge.
- Purely electric and purely magnetic species?

# The Four-Fluid Model

- We consider a plasma with four components:
  - Electrons: electric charge of $-q_e$
  - Ions: positive electric charge of $q_e$, large mass compared to electrons
  - Monopoles: magnetic charge of $q_m$
  - Antimonopoles: magnetic charge of $-q_m$, same mass as monopoles
- Each species has its own set of fluid equations and enters into Maxwell's equations through the charge and current densities.

# MHD Waves

To find waves in MHD, we need to find linear perturbations on top of a constant background.

For some quantity:

$$X_s = X_{s,0} + X_{s,1}\left(\boldsymbol{x}, t\right)$$

A constant background

Wave-like perturbation

$$X_{s,1}\left(\boldsymbol{x}, t\right) = X_{s,1} e^{i\left(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t\right)}$$

# MHD Waves

$$X_s = X_{s,0} + X_{s,1}\left(\boldsymbol{x}, t\right)$$

$$X_{s,1}\left(\boldsymbol{x}, t\right) = X_{s,1} e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)}$$

- For each species, use a perturbation of this form for each quantity, throw out terms that are higher than first order.
- Ions are massive, so we assume their density doesn't vary from the background in the time scale that the electron density changes.
- Assume quasi-neutrality so that the background fields vanish.

# MHD Waves

We define plasma frequencies for each component

Electrons:

$$\omega_{p,e}^2 = 4\pi \frac{q_e^2 n_{e,0}}{m_e}$$

Monopoles and antimonopoles:

$$\omega_{p,m}^2 = \omega_{p,+}^2 + \omega_{p,-}^2 = 4\pi \frac{q_m^2 n_{+,0}}{m_\pm} + 4\pi \frac{q_m^2 n_{-,0}}{m_\pm}$$

# MHD Waves

Using the assumptions for a cold plasma, the MHD equations reduce to these two vector equations

$$\boldsymbol{k} \times \boldsymbol{E}_1 = \left( \frac{\omega}{c} - \frac{\omega_{p,m}^2}{c\omega} \right) \boldsymbol{B}_1$$

$$\boldsymbol{k} \times \boldsymbol{B}_1 = \left( -\frac{\omega}{c} + \frac{\omega_{p,e}^2}{c\omega} \right) \boldsymbol{E}_1$$

# Electromagnetic Waves

EM waves, by definition, travel perpendicular to the fields. Using this, we obtain the dispersion relation

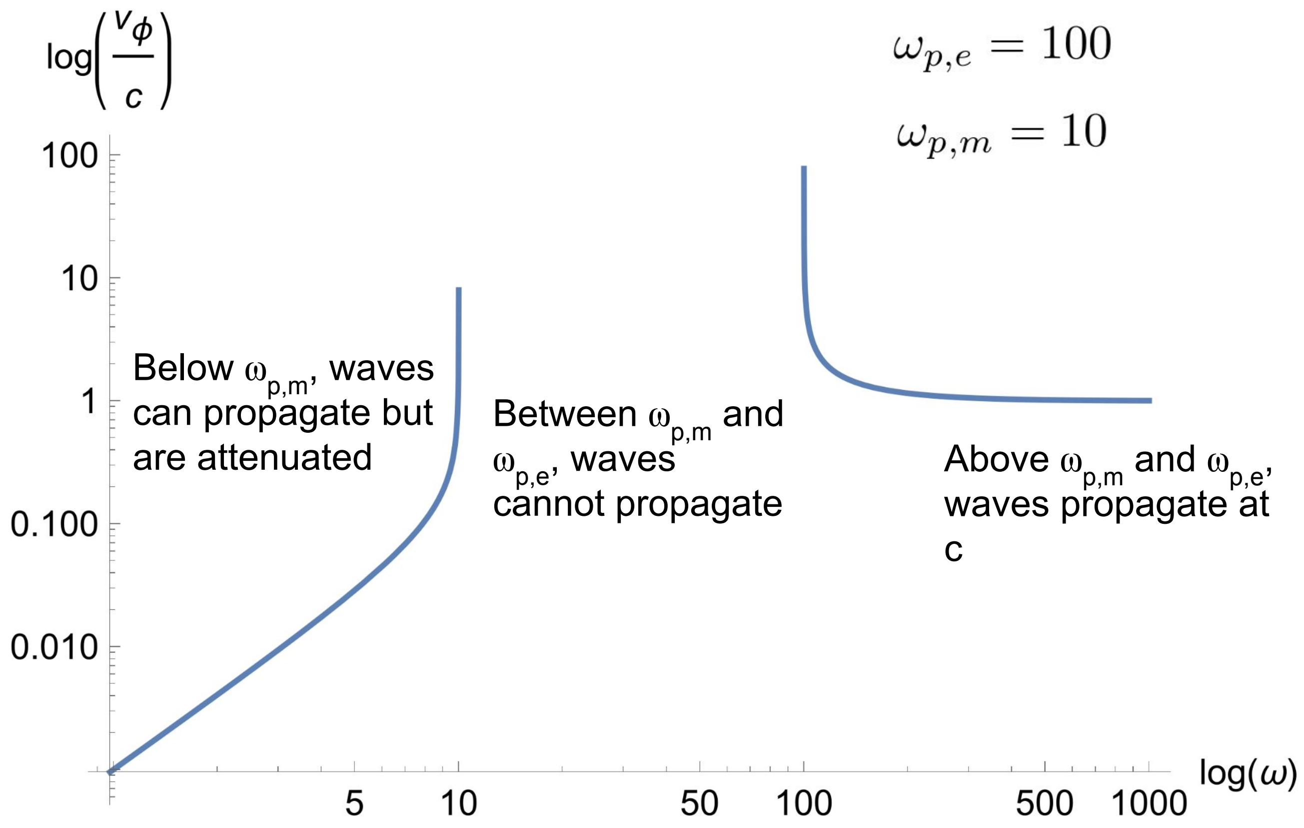
$$\frac{c^2k^2}{\omega^2} = \left(1 - \frac{\omega_{p,m}^2}{\omega^2}\right)\left(1 - \frac{\omega_{p,e}^2}{\omega^2}\right)$$

# Electromagnetic Waves

This yields the phase velocity

$$v_\phi = \frac{\omega}{k} = c\left[\left(1 - \frac{\omega_{p,m}^2}{\omega^2}\right)\left(1 - \frac{\omega_{p,e}^2}{\omega^2}\right)\right]^{-1/2}$$

# Possible Probe for Monopoles?



# Thank you!

Any questions?